Mathematical and 3D Mechanical System Simulation of Arm Robot Modelled as Double Pendulum

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Abstrak

Lengan robot telah banyak digunakan dan penelitian ekstensif terkait manipulator telah dilakukan. Permasalahan pada manipulator robot lengan adalah pengendalian dan stabilisasinya. Pemodelan matematis dapat membantu mengendalikan lengan robot dengan parameter yang mudah untuk ditentukan. Penelitian ini bertujuan untuk membandingkan simulasi pemodelan matematis dan simulasi sistem mekanik 3D manipulator yang dimodelkan dengan pendulum ganda. Simulasi dilakukan dengan bantuan MATLAB/Simulink dan MATLAB/Simscape Multibody. Metode penelitian meliputi: (i) Pembentukan persamaan Lagrange untuk manipulator, (ii) Membuat blok MATLAB/Simulink, (iii) Membuat blok MATLAB/Simscape Multibody, (iv) Mendefinisikan parameter matematis dan fisika dan (v) Menjalankan simulasi dan menganalisis hasil. Hasil dari penelitian ini menunjukkan simulasi matematis lengan robot berdasarkan persamaan Lagrange konsisten dalam bentuk gelombang sinus. Sedangkan untuk simulasi 3D hasil grafiknya tidak konsisten. Namun pada hasil simulasi 3D yang parameter lengan robotnya hampir sama, hasilnya menunjukkan tren tertutup.

Kata kunci: manipulator; pendulum ganda; *lagrange*; simulasi sistem mekanik

Abstract

Arm robot has been widely used and extesives researches related to manipulator were already been conducted.Problem with arm robot manipulator is the control and stabilization. The mathematical modelling can help to control arm robot with parameter which easy to manipulate. This study is aim to compare mathematical modelling simulation and 3D mechanical system simulation of manipulator modelled as double pendulum. The simulation conducted with help of MATLAB/Simulink and MATLAB/Simscape Multibody. Research methods consist of: (i) Lagrange equation forming for manipulator, (ii) Building MATLAB/Simulink block, (iii) Building MATLAB/Simscape Multibody block, (iv) Define mathematical and physical parameter and (v) Run simulation and analized the result.The result of this research is that the mathematical simulation result for arm robot based on Lagrange equation consistent in form of sine wave. Meanwhile for 3D simulation result is not a consistent graph. However, for 3D simulation result, where the arm robot parameters are similar the result showed closed trend.

Keywords: manipulator; double pendulum; lagrange; mechanical system simulation

1. Introduction

Robotics development includes many branch of sciences such as mechanical engineering, electronics engineering and artificial intelligent. Robotic arm is one of robot type which developed rapidly. Arm robot utilization varies widely from automotive and manufacture industries [1- 2] to health industries such as upper limb therapy aid [3]. Degree of freedom (DoF) varieties are developed by arm robot manufacturers which suit for various tasks. For simple task like pick and place task, fewer DoF is needed and for more difficult task like painting more DoF are needed [4].

Previous study related to arm robot has been done widely with various method, parameter, software aided and design. Many previous studies published about robotic arm analysis utilizing inverse and forward kinematics [5-6]. Meanwhile this study used Lagrangian formula to derived the arm robot mathematical model. Previous study also used lagrangian formula method to modelled and simulated two DoF arm robot [4]. The difference with this study is that this

study compare the mathematical simulation and 3D mechanical system simulation, meanwhile previous study not comparing both method [4-6].

Another study using software such as MATLAB as simulation aid for arm robot [7-9]. Some of those previous study using MATLAB/Virtual Reality as environtment illustration. This study also utilized MATLAB as simulation aid with MATLAB/Simulink and MATLAB/Simscape Multibody feature.

This study modelled two DoF arm manipulator as double pendulum and used Lagrange formula to derive the mathematical model approach. This study aim to compare the double pendulum oscillation between mathematical simulation and 3D mechanical system simulation. The purpose of this study is to provide two modeling simulation method and proof that MATLAB can do both simulation. As a powerfull software MATLAB can be utilized to simulate the mathematical modelling of physical system. MATLAB/Simulink feature facilitated reserachers to do the mathematical simulation. MATLAB also has feature to simulate the physical system using mechanical system which called MATLAB/Simscape Multibody; previously called MATLAB/SimMechanics. Arm robot frame modelled as body and other parts modelled as sensor or actuator.

2. Material dan methodology

Research methods of this study consists of five steps, i.e. (i) Lagrange equation forming for arm manipulator, (ii) Building MATLAB/Simulink block, (iii) Building MATLAB/Simscape Multibody block, (iv) Define mathematical and physical parameter and (v) Run simulation and analized the result. Figure 1 shows the sketch of arm robot used to derive the Lagrange equation. Arm robot divide into two link, named link I and link II. Link I is lower arm part and link II is upper arm part of the robot.
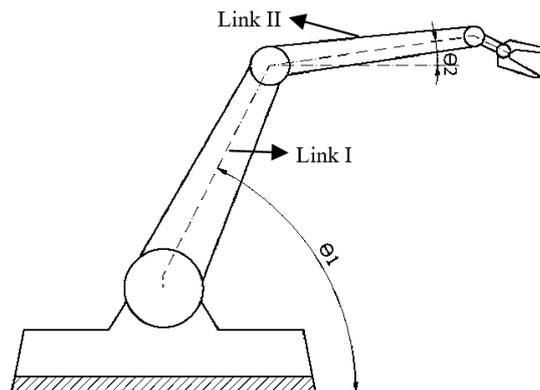

Figure 1. Arm robot illustration

Potential energy and kinetics energy equation are derived then concluded to total potential energy as well as total kinetics energy equation for link I and link II which represented in Equation (1) and Equation (2). Where V is potential energy total for link I and link II in Joule, θ_1 angle between x-axis and lower arm, θ_2 angle between x-axis and upper arm in radian, m_1 and m_2 are lower arm mass and upper arm mass in kg, b_1 and b_2 are centre of mass distance between in link I and link II in m, l1 is lower arm length in m.

$$V = -m_1 g b_1 cos\theta_1 + m_2 g(-b_2 cos(\theta_1 + \theta_2) - l_1 \cos\theta_1) \tag{1}$$

Equation (2) represent the total kinetics energy of link I and link II. Where T is kinetics energy total in Joule, I_1 and I_2 are inertia moment of link I and link II in kg.m2, $\dot{\theta}_1$ and $\dot{\theta}_2$ are angular velocity in rad/s.

$$T = \tfrac{1}{2} m_1 b_1^2 \, \dot{\theta}_1^2 + \tfrac{1}{2} I_1 \, \dot{\theta}_1^2 + \tfrac{1}{2} m_2 \left(b_2(\dot{\theta}_1 + \dot{\theta}_2) \right)^2 + 2b_2(\dot{\theta}_1 + \dot{\theta}_2)(l_1\dot{\theta}_1) \cos \theta_2 + \left(l_1\dot{\theta}_1 \right)^2 + \tfrac{1}{2} I_2 \left(\dot{\theta}_1 + \dot{\theta}_2 \right)^2 \qquad (2)$$

Equation (1) and Equation (2) can be simplified into Equation (3) to Equation (8).

$$H_0 = m_1 b_1^2 + I_1 \qquad (3)$$

$$H_1 = m_2 l_1 b_2 \qquad (4)$$

$$H_2 = m_2 l_1^2 + m_2 b_2^2 + I_2 \qquad (5)$$

$$H_3 = m_2 b_2^2 + I_2 \qquad (6)$$

$$G_1 = (m_1 b_1 + m_2 l_1)g \qquad (7)$$

$$G_2 = m_2 b_2 g \qquad (8)$$

where H_0 is low arm inertia around θ_1 axis, H_1 is term related to inertia of upper arm about θ_1 axis, H_2 is term related to inertia of lower arm about θ_1 axis, H_3 is inertia of lower arm about θ_2 axis, G_1 is relation of gravitational torque between link I and link II with θ_1 and G_2 is relation of gravitational torque between link I with θ_2. Equation (1) and Equation (2) can be written in simplified model related to inertia and gravitational torque and represented in Equation (9) and Equation (10).

$$T = \left(\frac{1}{2}H_0 + \frac{1}{2}H_2 + H_1\cos(\theta_2) \right)\dot{\theta}_1^2 + \frac{1}{2}H_3\dot{\theta}_2^2 + (H_3 + H_1\cos\theta_2)\dot{\theta}_1\dot{\theta}_2 \qquad (9)$$

$$V = -G_1 \cos(\theta_1) - G_2\cos(\theta_1 + \theta_2) \qquad (10)$$

Potentials and kinetics energy equation of arm robot the formed into Lagrange formula, Ł = T-V, which can be seen in Equation (11).

$$£ = \left(\tfrac{1}{2}J_0 + \tfrac{1}{2}J_2 + J_1\cos(\theta_2) \right)\dot{\theta}_1^2 + \tfrac{1}{2}J_3\dot{\theta}_2^2 + (J_3 + J_1\cos\theta_2)\dot{\theta}_1\dot{\theta}_2 + G_1 \cos(\theta_1) + G_2\cos(\theta_1 + \theta_2) \qquad (11)$$

The Lagrange formula can be written where torque (τ) involved, denoted in Equation (12) and Equation (13).

$$\frac{d}{dt}\left[\frac{\partial £}{\partial \dot{\theta}_1}\right] - \left[\frac{\partial £}{\partial \theta_1}\right] = \tau_1 \qquad (12)$$

$$\frac{d}{dt}\left[\frac{\partial £}{\partial \dot{\theta}_2}\right] - \left[\frac{\partial £}{\partial \theta_2}\right] = \tau_2 \qquad (13)$$

Equation (14) and Equation (15) concluded the final Lagrange formula for link I and link II. Where

$$-2H_1\sin\theta_2\dot{\theta}_1\dot{\theta}_2 + (H_0 + H_2 + H_1\cos\theta_2)\ddot{\theta}_1 - H_1 \sin \theta_2\ddot{\theta}_2^2 + (H_3 + H_1\cos\theta_2)\ddot{\theta}_2 + G_1\sin\theta_1 + G_2 \sin(\theta_1 + \theta_2) = \tau_1 \qquad (14)$$

$$H_3\ddot{\theta}_2 - H_1 \sin \theta_2\dot{\theta}_1\dot{\theta}_2 + (H_3 + H_1\cos\theta_2)\ddot{\theta}_1 + G_2 \sin(\theta_1 + \theta_2) = \tau_2 \qquad (15)$$

Equation (14) and Equation (15) then modified into matrices form which can be seen on Equation (16)

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} (J_0 + J_2 + 2J_1\cos(\theta_2)) & (J_3 + J_1\cos(\theta_2)) \\ (J_3 + J1\cos(\theta_2)) & (J_3) \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -2J_1\sin(\theta_2)\,\dot{\theta}_1\dot{\theta}_2 - J_1\sin(\theta_2)\dot{\theta}_2^2 \\ -J_1\sin(\theta_2)\dot{\theta}_1\dot{\theta}_2 \end{bmatrix} + $$

$$\begin{bmatrix} G_1\sin(\theta_1) + G_2\sin(\theta_1 + \theta_2) \\ G_2\sin(\theta_1 + \theta_2) \end{bmatrix} \tag{16}$$

Equation (16) is simplified into Equation (17) in the form q1 and q2 to make it easier in the simulation. Where q_1 is θ_1 and q_2 is $\theta_1 + \theta_2$.

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + \vec{g}(q) = \tau \tag{17}$$

Next step is building MATLAB/Simulink block which can be seen in Figure 2.
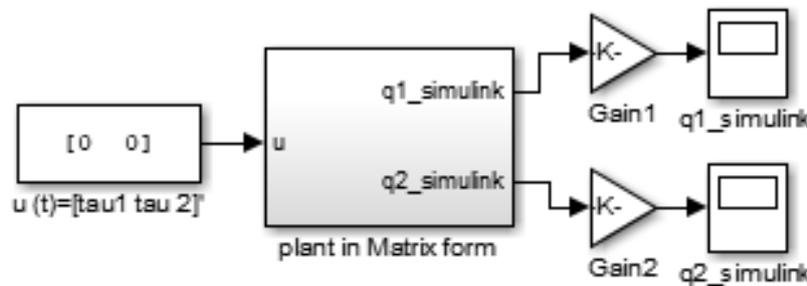

Figure 2. MATLAB/Simulink block for arm robot model

Arm robot mathematical model simulated in MATLAB/Simulink using matrices form which represented the plant system. Gain block showed in Figure 2 is used to convert q1 and q2 unit which in radian unit into degree unit. Parameter u(t) consist of tau1 and tau2 matrices are the arm robot input which denoted intorque value.

Figure 3 illustrated the next step i.e. MATLAB/Simscape block building.
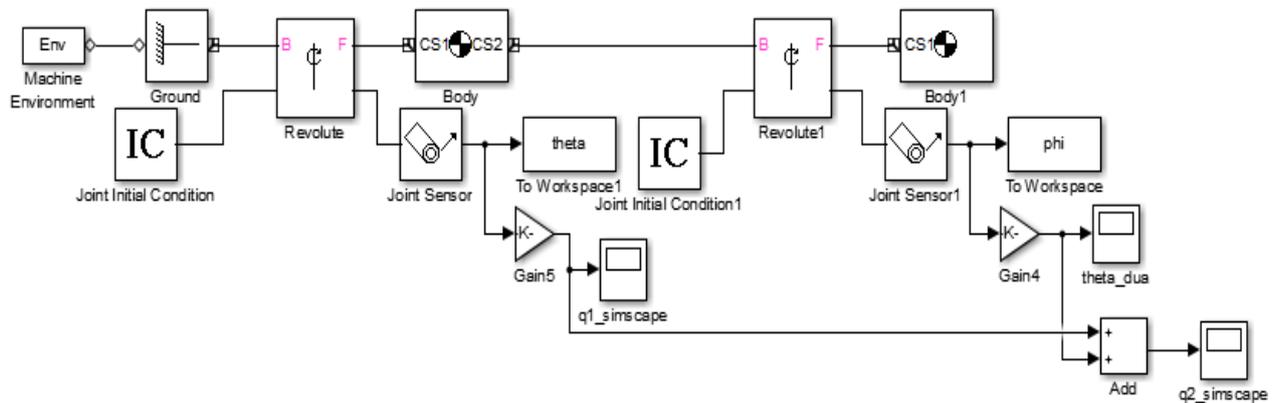

Figure 3. MATLAB/Simscape block for arm robot model

Body and body1 block represent the arm robot link I and link II. Joint initial condition block represent the motor torque input and joint sensor block represent motor position output q1 and q2.

After the MATLAB/Simulink block and MATLAB/Simscape block is arranged then the next step is to define the mathematical and physical parameter. Figure 4 shows the arm robot build in the present research. The arm robot moved using flex sensor and can be programmed to do object moving related tasks. The arm robot frame are 3D printed.
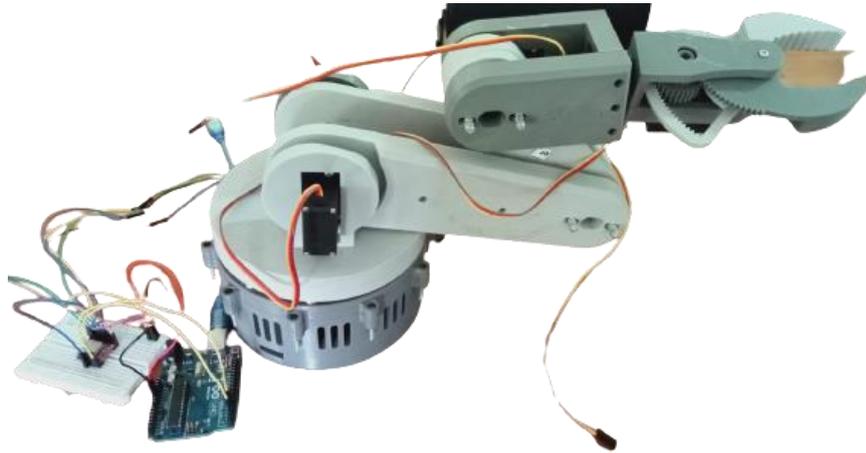

Figure 4. Arm robot build in the present research

This research used mathematical and physical parameter based on arm robot build in Figure 4 also compare with another arm robot by some manufacturers. Table 1 shows the mathematical and physical parameter used in the simulation. There are four arm robot models to be compared.

Table 1. Mathematical and physiscal parameter used in simulation

Parameter	Arm robot 1 [10]	Arm robot 2 [11]	Arm robot 3 [12]	Arm robot 4 (this research)
m_1 (kg)	10.68	0.6	25	0.75
m_2 (kg)	7.12	0.58	15	0.50
l_1 (m)	0.513	0.330	2.15	0.36
l_2(m)	0.342	0.299	1.4	0.28
b_1 (m)	0.257	0.165	1.08	0.18
b_2 (m)	0.171	0.086	0.7	0.14
I_1 (kg.m^2)	0.705	0.016	29.16	0.0243
I_2 (kg.m^2)	0.208	0.0043	7.35	0.0098

After simulation parameter being defined then last step is to simulate and analysis the result.

3. Results and Discussion

Arm robot simulation result describes the double pendulum oscillatory motion between mathematical equation and 3D simulation. Figure 5 shows the simulation result for arm robot 1.

Figure 5(a) shows that q_1 position peak for robot arm 1 3D simulation is at 56.9^0 on 3.2 s simulation time and q_1 position peak for mathematical simulation is at 57.9^0 on 1.96 s simulation time.

Figure 5(b) illustrated that for q_2 position initial oscillation, both 3D simulation and mathematical simulation start at the same angle. q_2 position peak for 3D simulation is at 90^0 on 0 s simulation time and q_2 position peak for mathematical simulation is at 90^0 on 0 s simulation time. Figure 6 shows the simulation result for arm robot 2 for q_1 and q_2 position.
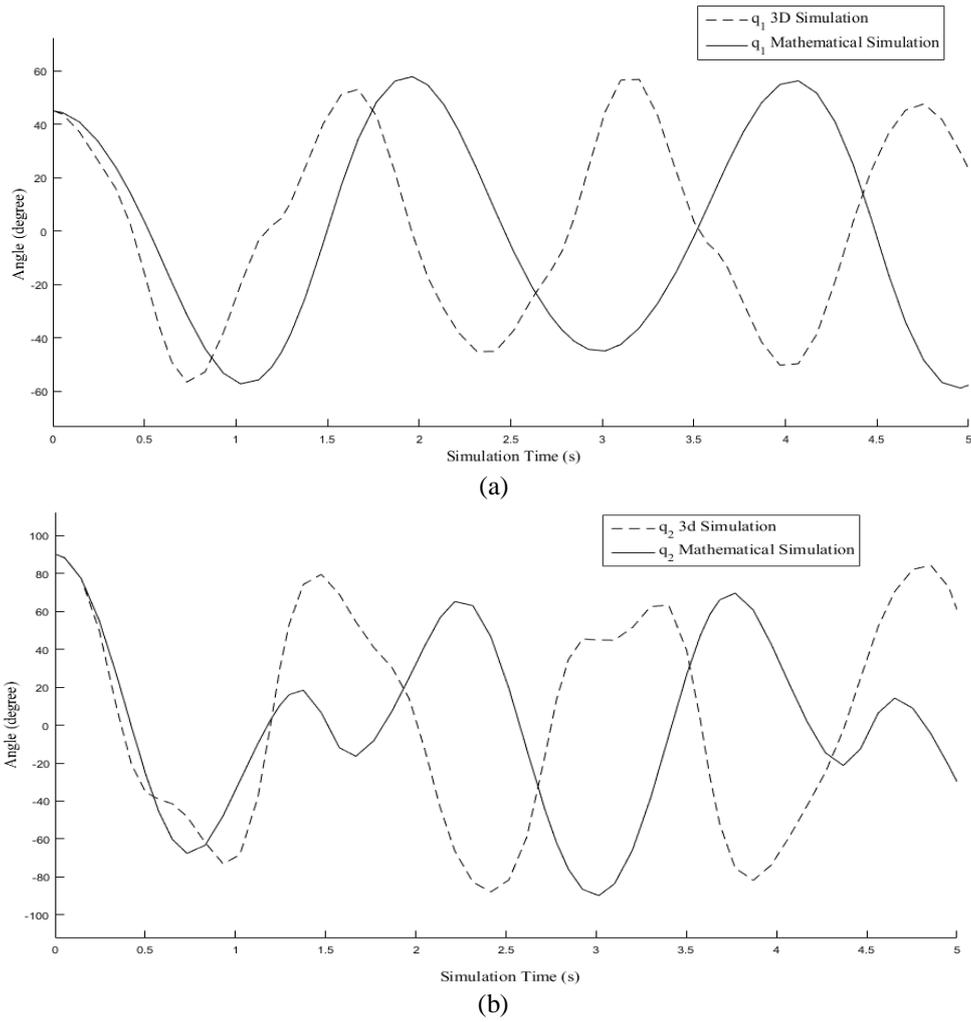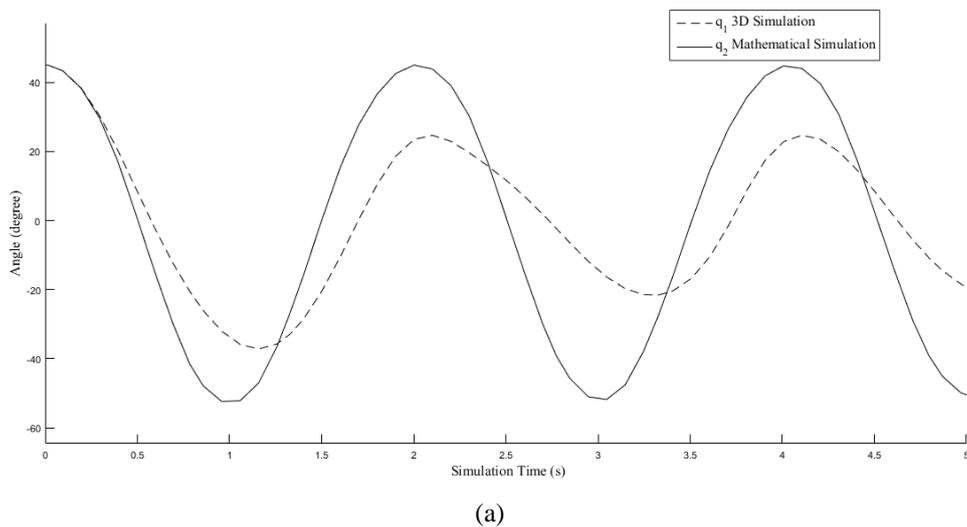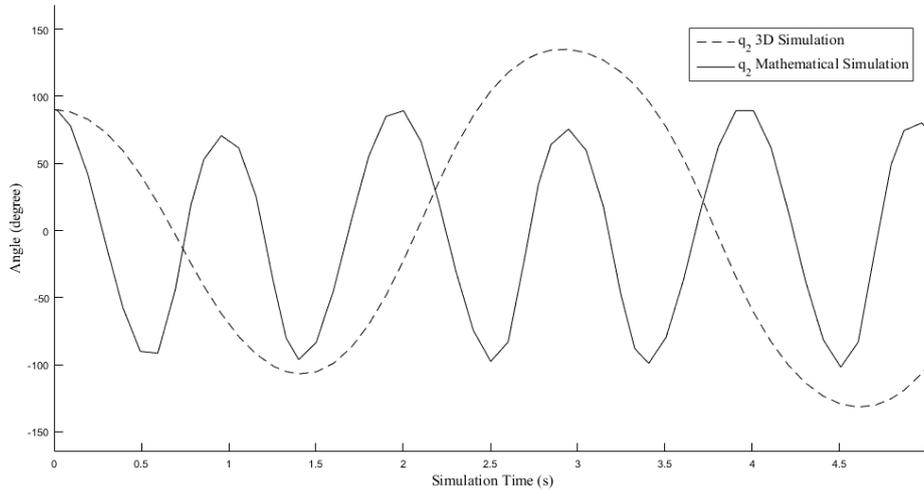
(a)

(b)

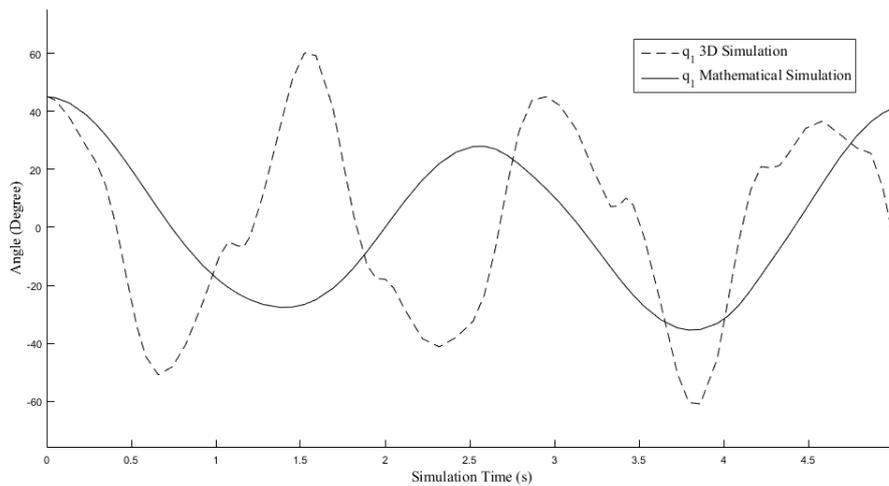**Figure 5.** Simulation result for robot arm 1: (a) q_1 position and (b) q_2 position

Figure 6(a) shows that q_1 position peak for robot arm 2 3D simulation is at 45^0 on 0 s simulation time and q_1 position peak for mathematical simulation is at 45^0 on 2.01 s simulation time. Figure 6(b) illustrated that q_2 position peak for 3D simulation is at 135^0 on 2.94 s simulation time and q_2 position peak for mathematical simulation is at 90^0 on 0 s simulation time. Figure 7 shows the simulation result for arm robot 3 for q_1 and q_2 position.
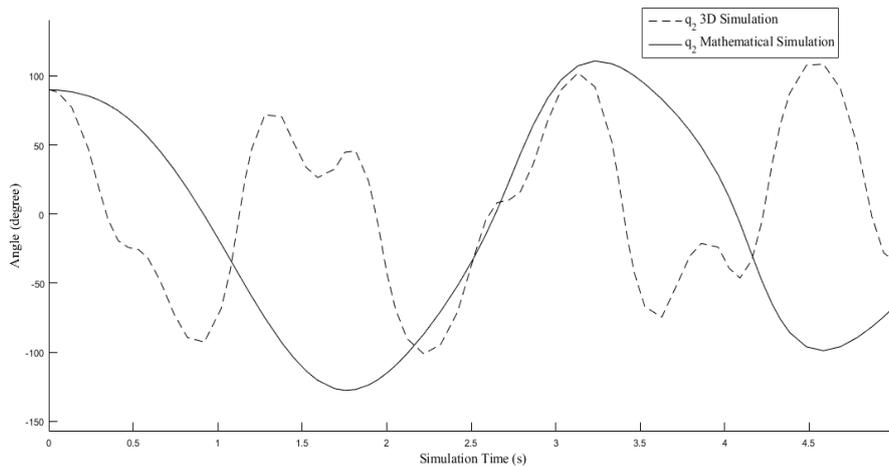

(a)

(b)

Figure 6. Simulation result for robot arm 2: (a) q_1 position and (b) q_2 position

Figure 7(a) shows that q_1 position peak for robot arm 3 3D simulation is at 60^0 on 1.5 s simulation time and q_1 position peak for mathematical simulation is at 45^0 on 0 s simulation time. Figure 7(b) illustrated that q_2 position peak for 3D simulation is at 108^0 on 4.58 s simulation time and q_2 position peak for mathematical simulation is at 110^0 on 3.23 s simulation time. Figure 8 shows the simulation result for arm robot in the present research for q_1 and q_2 position.
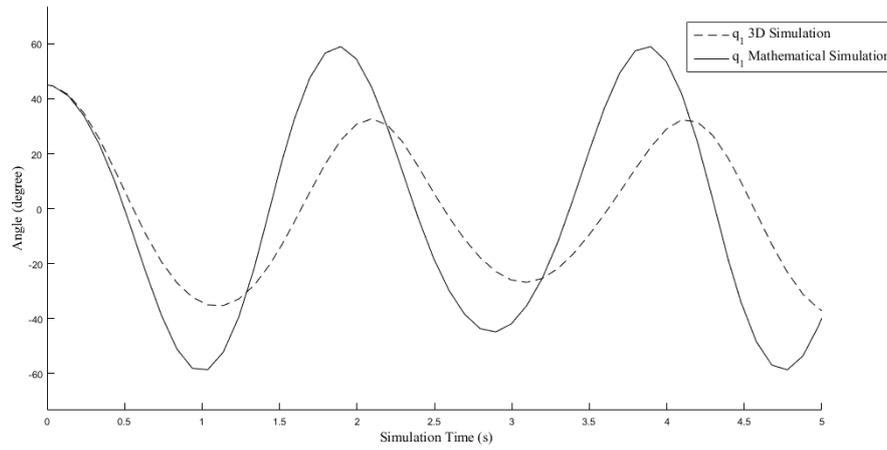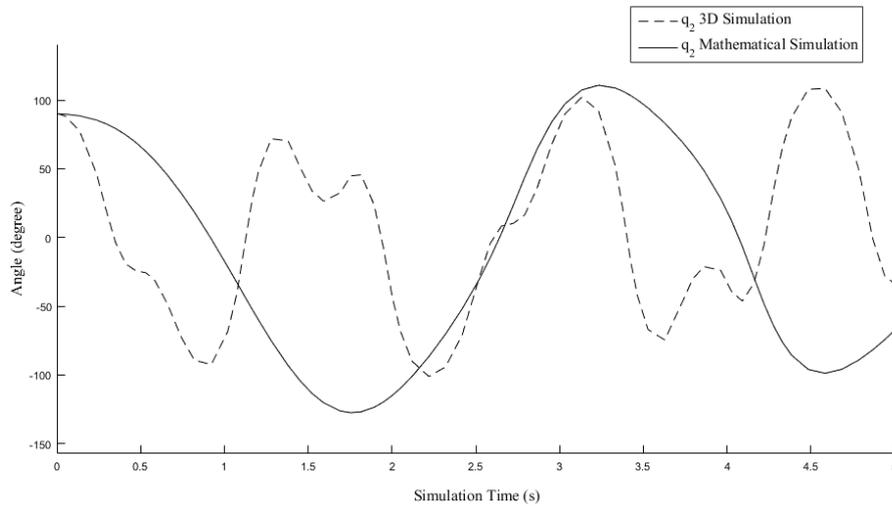

(a)

(b)

Figure 7. Simulation result for robot arm 3: (a) q_1 position and (b) q_2

(a)

(b)

Figure 8. Simulation result for robot present research: (a) q_1 position and (b) q_2

Figure 8(a) shows that q_1 position peak for robot arm in this present research 3D simulation is at 45^0 on 0 s simulation time and q_1 position peak for mathematical simulation is at 59^0 on 1.89 s simulation time. Figure 8(b) illustrated that q_2 position peak for 3D simulation is at 130^0 on 3.19 s simulation time and q_2 position peak for mathematical simulation is at 90^0 on 0 s simulation time. All q_1 position for mathematical simulation presented in Figure 9.
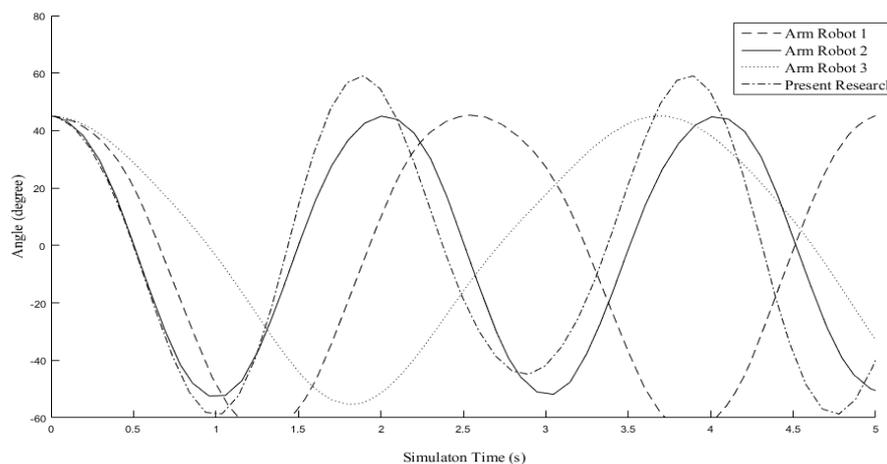

Figure 9. Comparison of q_1 position as result of mathematical simulation for all arm robot

Figure 9 illustrated that mathematical simulation for arm robot consistent in form of sine wave. The thing that differenciate the sine wave is the amplitude, wave length and frequency due to various arm robot parameter. Sine wave for arm robot 2 and present research are close to each other. Both arm robot 2 and present research have similar parameter in dimension and weight. Figure 10 shows all q_1 position for 3D simulation.
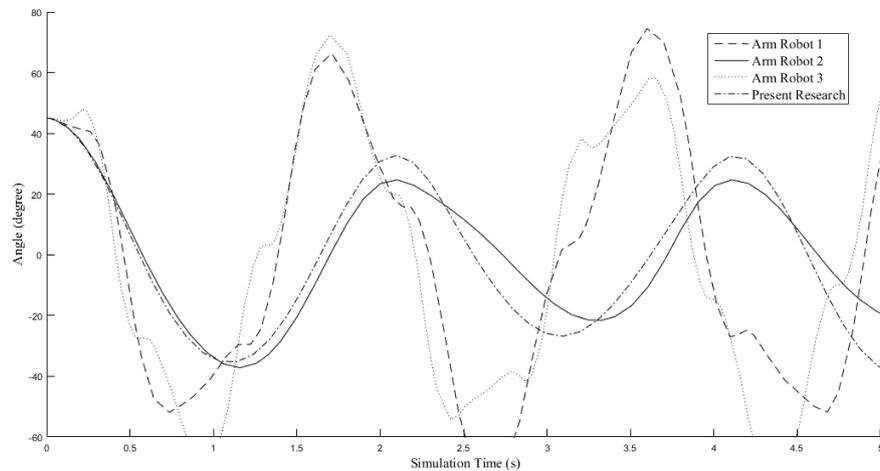

Figure 10. Comparison of q_1 position as result of 3D simulation for all arm robot

3D simulation results are not consistent to each other. However, arm robot with similar dimension and weight resulted in graphic which close to one another.

4. Conclusions

Both 3D system simulation and mathematical modelling simulation for arm robot can be a very usefull tool to simulate the robot environtment based on dimension and weight parameter. However, the mathematical modelling can be linearized to obtain the transfer function and derived the gain parameter in order to control the robot precisely. This result consistent with The research puspose which provide two simulation method using MATLAB. Further research can be done with mathematical modelling linearization and control parameter application into arm robot hardware [13-15].

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References

[1]   R. R. Aparnathi, "The Novel of Six axes Robotic Arm for Industrial Applications," *IAES Int. J. Robot. Autom.*, vol. 3, no. 3, Sep. 2014.

[2]   M. T. Rana and A. Roy, "Design and Construction of a Robotic Arm for Industrial Automation," *Int. J. Eng. Res. Technol.*, vol. 6, no. 5, pp. 919–922, 2017, [Online]. Available: www.ijert.org.

[3]   S. Cai, G. Huang, L. Huang, and L. Xie, "Kinematics analysis, design, and simulation of a dual-arm robot for upper limb physiotherapy," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 397, no. 1, 2018.

[4]   K. K. Megavarthini, K. Arun Kumar, P. Nagarajan, A. Mohammed Ismail, R. S. Karrthik, and S. Raghul Kumar,

"Formulation of lagrangian equation and dynamic analysis of 2 - DoF Industrial robot using Roboanalyzer," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 995, no. 1, p. 012013, Nov. 2020, doi: 10.1088/1757-899X/995/1/012013.

[5] T. Dewi, S. Nurmaini, P. Risma, Y. Oktarina, and M. Roriz, "Inverse kinematic analysis of 4 DOF pick and place arm robot manipulator using fuzzy logic controller," *Int. J. Electr. Comput. Eng.*, vol. 10, no. 2, p. 1376, Apr. 2020,

[6] H. M. Yudha, T. Dewi, P. Risma, and Y. Oktarina, "Arm Robot Manipulator Design and Control for Trajectory Tracking; a Review," *Proceeding Electr. Eng. Comput. Sci. Informatics*, vol. 5, no. 1, pp. 1–6, Nov. 2018,

[7] P. Sutyasadi and M. B. Wicaksono, "Joint control of a robotic arm using particle swarm optimization based H2/H∞ robust control on arduino," *TELKOMNIKA.*, vol. 18, no. 2, p. 1021, Apr. 2020,

[8] R. R. Serrezuela, A. F. C. Chavarro, M. A. T. Cardoso, A. L. Toquica, and L. F. O. Martinez, "Kinematic modelling of a robotic arm manipulator using MATLAB," *ARPN J. Eng. Appl. Sci.*, vol. 12, no. 7, pp. 2037–2045, 2017.

[9] H. A. E. G. Amr Nasr A. E. Gaber, Sohair F. Rezeka, "Design and Position Control of Arm Manipulator; Experimentally and in MATLAB Sim Mechanics," *Int. J. Eng. Res. Technol.*, vol. 5, no. 8, pp. 352–359, 2016, [Online]. Available: www.ijert.org.

[10] Franka Emika, "Data Sheet Robot Arm & Control," 2020. [Online]. Available: https://s3-eu-central-1.amazonaws.com/franka-de-uploads/uploads/Datasheet-EN.pdf.

[11] Crustcrawler Inc, "AX-12A / AX18A Smart Robotic Arm Specifications / Advantages," 2015.

[12] K. Heavy and R. Div, "Specification of Robot," 2014.

[13] Yingjie Tang and Xinwen Li, Simulation research of manipulator control system based on solidworks and simmechanics. Academic Journal of Computing and Information Science; 2017: 1(01): pp. 1-9.

[14] M.T. Trojnacki and S. Nycz., Modelling of mechanism using simmechanics toolbox of MATLAB/Simulink. International Journal of Applied Mechanics and Engineering. 2010; 15(03): pp. 875-883

[15] R. Halicioglu, L.C. Dugler and A.T. Bozdana, Modelling and simulation based on MATLAB/Simulink: A press mechanism. IOP Conference series; 2014; 012053.